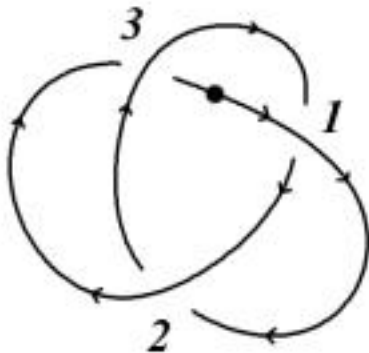


Virtual Knots via the description of Gauss Codes

Jay Patel

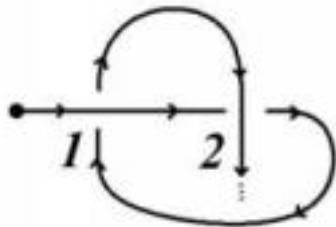
June 29, 2020

What is a Gauss Code?



01-U2-03-U1-02-U3-

What *else* is a Gauss Code?



01+U2+U1+02+

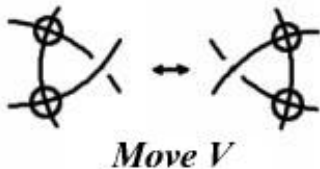
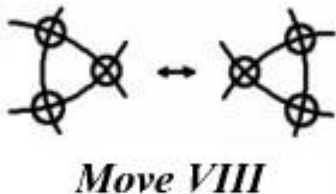
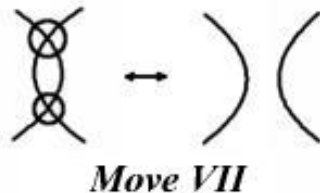
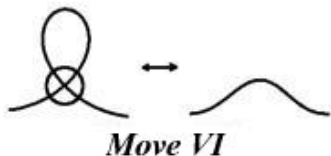
Virtual Knots in particular

The knot in the slide above can't be closed without introducing another crossing, but we can be brave and just sweep that issue under the rug by calling this new crossing "virtual" and we still have something that resembles a knot that we call a virtual knot.

Theorem

Every Gauss Code gives rise to a (tame) virtual knot and every (tame) virtual knot diagram can be turned into a Gauss Code

Brief review of Reidemeister moves for virtual knots



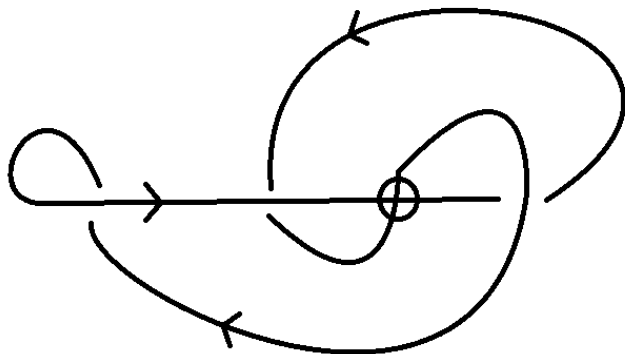
Trying an example

Example (Try to draw this Gauss Code and determine if it is a classical or virtual knot)

$O1+O2-U3+U2-O3+U1+$

Example illustrated in MS Paint

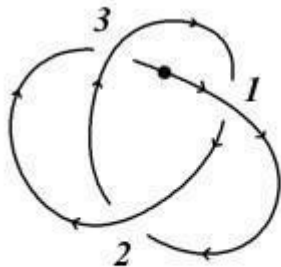
Figure: The classical crossings are 1,2,3 in order from right to left



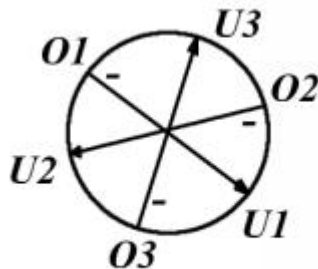
Invariance of Gauss Codes

- They aren't invariant under Reidemeister I and II because both of them reduce the number of crossings in the diagram and thus reduce the amount of terms in the Gauss Code
- Reidemeister III would permute the order of the code in a non-cyclic way (at least in the general case) and thus a code isn't necessarily invariant under it
- These facts about these codes greatly reduce the practicality of using Gauss codes for any real use other than just communicating a diagram to someone in a compact way

Gauss Diagrams come to the rescue



01-U2-03-U1-02-U3-



Gauss Diagram

Reidemeister moves on Gauss diagrams 1/2



Reidemeister Type I move



Reidemeister Type II move

Reidemeister moves on Gauss diagrams 2/2



Reidemeister Type III Move

Some things about Gauss Diagrams to keep in mind

- When drawing a Gauss Diagram, virtual crossings are to be omitted
- Only the “classical” Reidemeister moves appear on the Gauss Diagram while virtual moves are taken into account as you simply add the necessary virtual crossings you need at the end
- So, when you translate a knot diagram to a Gauss diagram, use Reidemeister moves, and then translate back to knot diagram (and add the necessary virtual crossings), you have effectively performed some virtual Reidemeister moves as well

Additional interesting information 1/3

Some particularly clever people may think up of a couple more combinations of virtual Reidemeister moves that involve moving two real crossings past a virtual crossing. But, unfortunately, these moves are not correct at all (the degree to which can be seen in a bit)

Additional interesting information 2/3

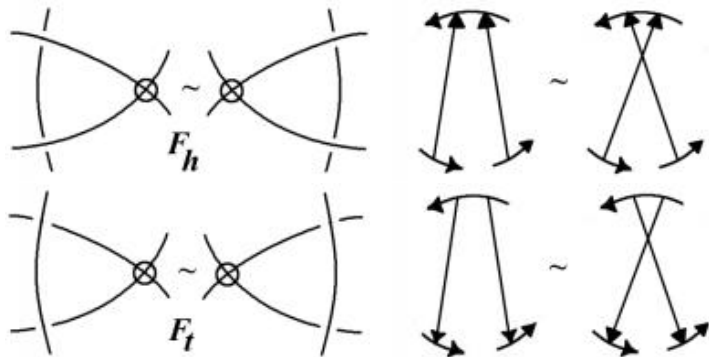


Figure: Here are the forbidden moves and their corresponding moves on the Gauss Diagram

Additional interesting information 3/3

These moves are forbidden for a very good reason:

Theorem

Any two Gauss Diagrams are equivalent under the Reidemeister moves \cup the two forbidden moves

Proof.

See the Sam Nelson paper in the References below

I have stolen most of the figures and notation from the following page introducing virtual knots by Sam Nelson:

<https://www1.cmc.edu/pages/faculty/VNelson/vknots.html>

Other than that, I used the following papers as references:

- Goussarov, Mikhail, Michael Polyak, and Oleg Viro. "Finite-type invariants of classical and virtual knots." *Topology* 39.5 (2000): 1045-1068.
- Nelson, Sam. "Unknotting virtual knots with Gauss diagram forbidden moves." *Journal of Knot Theory and Its Ramifications* 10.06 (2001): 931-935.